Chapter Test

2 a)

$$nC_r = {}_{n-1}C_{r-1} + {}_{n-1}C_{r-1}$$

$${}_{10}C_7 + {}_{10}C_8 = {}_{(10+1)}C_8.$$

$$= {}_{11}C_8.$$

6 b).

$${}_{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!}$$

$$= \frac{20 \times 19 \times 18 \times 17!}{17!}$$

$$= 20 \times 19 \times 18$$

$$= 6840 \text{ ways}.$$

3 a) $(3x-4)^4$

$$= 1(3x)^4 + 4(3x)^3(-4) + 6(3x)^2(-4)^2 + 4(3x)(-4)^3 + 1(-4)^4.$$

$$= 81x^4 + 4(27x^3)(-4) + 6(9x^2)(16) + 4(3x)(-64) + 256.$$

$$= 81x^4 - 432x^3 + 864x^2 - 768x + 256.$$

4. a) $(8x-3)^5$

$$(8x-3)^5 = \sum_{r=0}^{5} {}_5C_r (8x)^{5-r}(3^r).$$

$$= {}_5C_0(8x)^5 + {}_5C_1(8x)^4(3) + {}_5C_2(8x)^3(3^2) + {}_5C_3(8x)^2(3^3) + {}_5C_4(8x)(3^4) + {}_5C_5(3^5).$$

$$= 32768x^5 + 61440(-1)x^4 + 46080x^3 - 17280x^2 + 3240x - 243$$

$$= 32768x^5 - 61440x^4 + 46080x^3 - 17280x^2 + 3240x - 243.$$

a) $${}_{20}C_3 = \frac{20!}{3!\,17!} = \frac{20 \times 18 \times 19}{3 \times 2 \times 1}$$

$$= 1140 \text{ ways}.$$

c) answers in a) and b) should not be the same. because in a combination, the order of selection of subsets is not a factor whereas in permutation the order of selection is a factor.

5.1

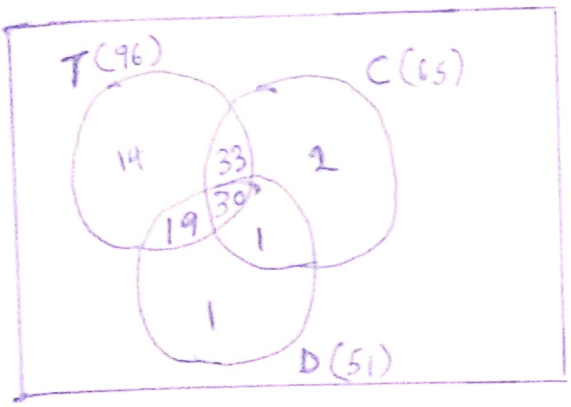
a) $A \cup B \Rightarrow R_2, R_3, R_5, R_6, R_8$.

b) $B \cap C \Rightarrow R_7, R_8$

c) $A \cap C \Rightarrow R_5, R_8$.

d) $B \cup S \Rightarrow R_3, R_6, R_7, R_8, R_1$.

3. b)



a) $100 - (14+2+1) - (30+33+19)$

$= 100 - 17 - 82$

$= 100 - 17 - 82$

$= 100 - 99$

$= 1\%$ of households were not included in these survey results.

5.2

4. f) ${}_{15}C_{11} = \frac{15!}{11!(15-11)!}$

$= \frac{15!}{11!\,4!}$

$= \frac{15 \times 14 \times 13 \times 12 \times 11!}{11!\,4!}$

$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$

$= 1365.$

5.3

10. ${}_5C_4 = \frac{5!}{4!} = \frac{5\times 4!}{4!} = 5.$

${}_7C_4 = \frac{7!}{4!3!} = \frac{7\times 6\times 5\times 4!}{4!\times 3\times 2\times 1}$

$= 35.$

ways to pick 8 questions $= 5\times 35$

$= 175$ ways.

Ways to pick 8 out of 12 questions.

$= 35 + 175$

$= 210$ ways.

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18. ${}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$

$= (15\times 14\times 13\times 12\times 11)/(1\times 2\times 3\times 4\times 5) \times (10\times 9\times 8\times 7\times 6)/(1\times 2\times 3\times 4\times 5)$

$= 3003 \times 252$

$= 756756$ ways

5.4

15 a) $(x+y)^6$

$= 1(x^6) + 6(x^5y) + 15(x^4y^2) + 20x^3y^3 + 15(x^2y^4) + 6x^2y^5 + y^6.$

$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^2y^5 + y^6.$

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19. $\left(\frac{1}{x^2} + 2x\right)^5$

$= 1\left(\frac{1}{x^2}\right)^5 + 5\left(\frac{1}{x^2}\right)^4(2x) + 10\left(\frac{1}{x^2}\right)^3(2x)^2 + 10\left(\frac{1}{x^2}\right)^2(2x)^3 + 5\left(\frac{1}{x^2}\right)(2x)^4 + (2x)^5.$

$= \frac{1}{x^{10}} + \frac{10}{x^7} + \frac{40}{x^4} + \frac{80}{x} + 80x^2 + 32x^5$

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23. 1024 to

$1024x^{10} - 3840x^8 + 5760x^6 - 4320x^4 + 1620x^2 - 243.$

There are six terms, so the exponent must be 5.

$1024x^{10} = (4x^2)^5$

$\therefore a = 4x^2.$

$-(+b)^5 = -243$

$b = \sqrt[5]{243} = 3.$

$b = -3$

expansion $(4x^2 - 3)^5$

5.4 The Binomial Theorem.

1. e) $(a+b)^5$

$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$.

4. $(a+b)^6$

Number of terms $= 6+1$

$= 7$ terms.

Apply, Solve, Communicate.

5. a) ${}_9C_0 + {}_9C_1 + \dots + {}_9C_9 = 2^9$

$= 512$.

6. $\sum_{r=0}^{n} {}_nC_r = 16384$.

$a^n = 16384$.

$2^n = 16384$

$2^n = 2^{14}$

$n = 14$

15 $(25x^2 + 30xy + 9y^2)^3$.

factor $25x^2 + 30xy + 9y^2$

$a+b = 30$

$a \times b = 225$

$25x^2 + 15xy + 15xy + 9y^2$

$5x(5x+3y) + 3y(5x+3y)$

$(5x+3y)(5x+3y)$

$((5x+3y)^2)^3$.

$(5x+3y)^6$.

expand.

$= (5x)^6 + 6(5x)^5(3y) + 15(5x)^4(3y)^2 + 20(5x)^3(3y)^3 + 15(5x)^2(3y)^4 + 6(5x)(3y)^5 + (3y)^6$.

$= 15625x^6 + 56250x^5y + 84375x^4y^2 + 67500x^3y^3 + 30375x^2y^4 + 7290xy^5 + 729y^6$.

$= 15625x^6 + 56250x^5y + 84375x^4y^2 + 67500x^3y^3 + 30375x^2y^4 + 7290xy^5 + 729y^6$